Some Common Mathematical Symbols and Abbreviations (with History)

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Binary Relations

$=$ (the **equals sign**) means "is the same as" and was first introduced in the 1557 book *The Whetstone of Witte* by physician and mathematician Robert Recorde (c. 1510–1558). He wrote, "I will sette as I doe often in woorke use, a paire of parralles, or Gemowe lines of one lengthe, thus: $=\!=\!=\!=$, bicause noe 2 thynges can be moare equalle." (Recorde's equals sign was significantly longer than the one in modern usage.)

$<$ (the **less than sign**) means "is strictly less than", and $>$ (the **greater than sign**) means "is strictly greater than". These first appeared in *Artis Analyticae Praxis ad Aequationes Algebraicas Resolvendas* ("The Analytical Arts Applied to Solving Algebraic Equations") by mathematician and astronomer Thomas Harriot (1560–1621), which was published posthumously in 1631.

Pierre Bouguer (1698-1758) later refined these to $\leq$ ("is less than or equals") and $\geq$ ("is greater than or equals") in 1734. Bouger is sometimes called "the father of naval architecture" due to his foundational work in the theory of naval navigation.

$:=$ (the **equal by definition sign**) means "is equal by definition to". This is a common alternate form of the symbol "$=_{\text{Def}}$", which first appeared in the 1894 book *Logica Matematica* by the logician Cesare Burali-Forti (1861–1931). Other common alternate forms of the symbol "$=_{\text{Def}}$" include "$\stackrel{\text{def}}{=}$" and "$\equiv$", the latter being especially common in applied mathematics.

$\doteq$ (the **approximately equals sign**) means "is nearly equal to" and was first used in 1875 by mathematician Anton Steinhauser (1802–1890) in his *Lehrbuch der Mathematik*. (This symbol was also briefly used in 1832 by geometer Farkas Wolfgang Bolyai (1775–1856) to signify "equal by definition".) Other modern symbols for "approximately equals" include "$\approx$" (read as "is approximately equal to"), "$\cong$" (read as "is congruent to"), "$\simeq$" (read as "is similar to"), "$\asymp$" (read as "is asymptotically equal to"), and "$\propto$" (read as "is proportional to"). Usage varies, and these are sometimes used to denote varying degrees of "approximate equality" within some context.

Some Symbols from Mathematical Logic

$\therefore$ (**three dots**) means "therefore" and first appeared in print in the 1659 book *Teusche Algebra* ("Teach Yourself Algebra") by mathematician Johann Rahn (1622–1676). (*Teusche Algebra* also contains the first use of the **obelus**, "$\div$", to denote division.)

$\because$ (**upside-down dots**) means "because" and seems to have first appeared in the 1805 book *The Gentleman's Mathematical Companion*. However, it is much more common (and less ambiguous) to just abbreviate "because" as "b/c".

$\ni$ (the **such that sign**) means "under the condition that" and first appeared in the 1906 edition of *Formulaire de mathematiqués* by the logician Giuseppe Peano (1858–1932). However, it is much more common (and less ambiguous) to just abbreviate "such that" as "s.t.".

There are two good reasons to avoid using "$\ni$" in place of "such that". First of all, the abbreviation "s.t." is significantly more suggestive of its meaning than is "$\ni$". Perhaps more importantly, though, is that it has become increasingly common for the symbol "$\ni$" to mean "contains as an element", which is a logical extension of the usage of the unquestionably standard symbol "$\in$" to mean "is contained as an element in".

$\Rightarrow$ (the **implies sign**) means "logically implies that", and $\Leftarrow$ (the **is implied by sign**) means "is logically implied by". Both have an unclear historical origin. (E.g., "if it's raining, then it's pouring" is equivalent to saying "it's raining $\Rightarrow$ it's pouring.")

$\iff$ (the **iff** symbol) means "if and only if" and is used to connect logically equivalent statements. (E.g., "it's raining iff it's really humid" means simultaneously that "if it's raining, then it's really humid" and that "if it's really humid, then it's raining". In other words, the statement "it's raining $\iff$ it's really humid" means simultaneously that "it's raining $\Rightarrow$ it's really humid" and "it's raining $\Leftarrow$ it's really humid".)

The abbreviation "iff" is attributed to the mathematician Paul Halmos (1916–2006).

$\forall$ (the **universal quantifier**) means "for all" and was first used in the 1935 publication *Untersuchungen ueber das logische Schliessen* ("Investigations on Logical Reasoning") by logician Gerhard Gentzen (1909–1945). He called it the *All-Zeichen* ("all character") by analogy to the symbol "$\exists$" , which means "there exists".

$\exists$ (the **existential quantifier**) means "there exists" and was first used in the 1897 edition of *Formulaire de mathematiqués* by logician Giuseppe Peano (1858–1932).

$\square$ (the **Halmos tombstone** or **Halmos symbol**) means "Q.E.D.", which is an abbreviation for the Latin phrase *quod erat demonstrandum* ("which was to be proven"). "Q.E.D." has been the most common way to symbolize the end of a logical argument for many centuries, but the modern convention of the "tombstone" is now generally preferred because it is easier to write and is also visually more compact.

The symbol "$\square$" was first made popular by mathematician Paul Halmos (1916–2006).

Some Notation from Set Theory

- $\subset$ (the **is included in sign**) means "is a subset of" and $\supset$ (the **includes sign**) means "has as a subset". Both symbols were introduced in the 1890 book *Vorlesungen über die Algebra der Logik* ("Lectures on the Algebra of the Logic") by logician Ernst Schröder (1841–1902).

- $\in$ (the **is in sign**) means "is an element of" and first appeared in the 1895 edition of *Formulaire de mathematiqués* by logician Giuseppe Peano (1858-1932). Peano originally used the Greek letter "ϵ" (viz. the first letter of the Latin word *est* for "is"), and it was the great logician and philosopher Betrand Russell (1872–1970) who introduced the modern stylized version of this symbol in his 1903 book *Principles of Mathematics*.

  It is also common to use the symbol "$\ni$" to mean "contains as an element", which is not to be confused with the more archaic usage of "$\ni$" to mean "such that".

- $\cup$ (the **union sign**) means "take the elements that are in either set", and $\cap$ (the **intersection sign**) means "take the elements that the two sets have in common". They were introduced in the 1888 book *Calcolo geometrico secondo l'Ausdehnungslehre di H. Grassmann preceduto dalle operazioni della logica deduttiva* ("Geometric Calculus based upon the teachings of H. Grassman, preceded by the operations of deductive logic") by logician Giuseppe Peano (1858–1932).

- $\emptyset$ (the **null set** or **empty set**) means "the set without any elements in it" and was first used in the 1939 book *Éléments de mathématique* by Nicolas Bourbaki. (Bourbaki is the collective pseudonym for a group of primarily European mathematicians who have written many mathematics books together.) It was borrowed simultaneously from the Norwegian, Danish and Faroese alphabets by group member André Weil (1906–1998).

- ∞ (**infinity**) denotes "a quantity or number of arbitrarily large magnitude" and first appeared in print in the 1655 *De Sectionibus Conicus* ("Tract on Conic Sections") by mathematician John Wallis (1616–1703).

  Possible explanations for Wallis' choice of "∞" include its resemblance to the symbol "*oo*" (used by ancient Romans to denote the number 1000), to the final letter of the Greek alphabet ω (used symbolically to mean the "final" number), and to the ease with which this simple curve (called a "lemniscate") can be endlessly traversed.

Some Important Numbers in Mathematics

- π (the **ratio of the circumference to the diameter of a circle**) denotes the number 3.141592653589..., and was first used by mathematician William Jones (1675–1749) in his 1706 book *Synopsis palmariorum mathesios* ("A New Introduction to Mathematics"). It was then the great mathematician Leonhard Euler (1707–1783) who popularized the use of π to denote this number in his 1748 book *Introductio in Analysin*

Infinitorum. (It is speculated that Jones chose the letter "π" because "π" is the first letter in the Greek word *perimetron* ($\pi\epsilon\rho\iota\mu\epsilon\tau\rho o\nu$), which roughly means "around".)

$e = \lim_{n\to\infty}(1+\frac{1}{n})^n$ (the **natural logarithm base**, also sometimes called **Euler's number**) denotes the number 2.718281828459..., and was first used by Leonhard Euler (1707–1783) in the manuscript *Meditatio in Experimenta explosione tormentorum nuper instituta* ("Meditation on experiments made recently on the firing of cannon"), which was written when Euler was only 21 years old. (It is speculated that Euler chose "e" because "e" is the first letter in the word "exponential".)

The mathematician Edmund Landau (1877–1938) once wrote that, "The letter e may now no longer be used to denote anything other than this positive universal constant."

$i = \sqrt{-1}$ (the **imaginary unit**) was first used by Leonhard Euler (1707–1783) in his 1777 memoir *Institutionum calculi integralis* ("Foundations of Integral Calculus").

The five most important numbers in mathematics are widely considered to be (roughly in order) 0, 1, i, π, and e, which are remarkably linked by the equation $e^{i\pi}+1=0$.

$\gamma = \lim_{n\to\infty}(\sum_{k=1}^{n}\frac{1}{k} - \ln n)$ (the **Euler-Mascheroni constant**, also known as just **Euler's constant**), denotes the number 0.577215664901..., and was first used by geometer Lorenzo Mascheroni (1750–1800) in his 1792 book *Adnotationes ad Euleri Calculum Integralem* ("Annotations to Euler's Integral Calculus").

The number γ is widely considered to be the sixth most important important number in mathematics due to its frequent appearance in formulas from number theory and applied mathematics. However, no one knows whether γ is even an irrational number.

$\phi = \frac{1}{2}(1+\sqrt{5})$ (the **golden ratio**) denotes the number 1.618033988749.... Its use was first attributed to the American Mathematician Mr. Mark Barr in *The Curves of Life: Being an Account of Spiral Formations and Their Application to Growth in Nature, to Science, and to Art: With Special Reference to the Manuscripts of Leonardo da Vinci (1914)* by Sir Theodore Andrea Cook (1867–1928):

> The symbol ϕ was given to this proportion partly because it has a familiar sound to those who wrestle constantly with π and partly because it is the 1st letter of the name of Pheidias, in whose sculpture this number is seen to prevail when the distance between salient points are measured.

The number ϕ is also often called the "divine proportion" or the "golden proportion", and it has been recognized since antiquity as an especially aesthetically pleasing ratio for the side lengths of a rectangle. Such a rectangle is called a "golden rectangle".

Appendix: Common Latin Abbreviations and Phrases

i.e. (*id est*) means "that is" or "in other words". (It is used to paraphrase a statement that was just made, **not** to mean "for example", and is **always** followed by a comma.)

e.g. (*exempli gratia*) means "for example". (It is usually used to give an example of a statement that was just made and is **always** followed by a comma.)

viz. (*videlicet*) means "namely" or "more specifically". (It is used to clarify a statement that was just made by providing more information and is **never** followed by a comma.)

etc. (*et cetera*) means "and so forth" or "and so on". (It is used to suggest that the reader should infer further examples from a list has already been started and is **usually not** followed by a comma.)

et al. (*et alii*) means "and others". (It is used in place of listing multiple authors past the first and is **never** followed by a comma.) The abbreviation "et al." can also be used in place of *et alibi*, which means "and elsewhere".

cf. (*conferre*) means "compare to" or "see also". (It is used either to draw a comparison or to refer the reader to somewhere else that they can find more information, and it is **never** followed by a comma.)

q.v. (*quod vide*) means "which see" or "go look it up if you're interested". (It is used to cross-reference a different written work or a different part of the same written work, and it is **never** followed by a comma.) The plural form of "q.v." is "q.q."

v.s. (*vide supra*) means "see above". (It is used to imply that more information can be found before the current point in a written work and is **never** followed by a comma.)

N.B. (*Nota Bene*) means "note well" or "pay attention to the following". (It is used to imply that the wise reader will pay especially careful attention to what follows and is **never** followed by a comma. Cf. the abbreviation "verb. sap.")

vs. (*versus*) means "against" or "in contrast to". (It is used to contrast two things and is **never** followed by a comma.)

c. (*circa*) means "around" or "near". (It is used when giving an approximation, usually for a date, and is **never** followed by a comma.) The abbreviation "c." is also commonly written as "ca.", "cir.", or "circ."

ex lib. (*ex libris*) means "from the library of". (It is used to indicate ownership of a book and is **never** followed by a comma.).

- *vice versa* means "the other way around" and is used to indicate that an implication can logically be reversed. (This is sometimes abbreviated as "v.v.")
- *a fortiori* means "from the stronger" or "more importantly".
- *a priori* means "from before the fact" and refers to reasoning that is done while an event still has yet to happen (such as the impending page break in this list).

- *a posteriori* means "from after the fact" and refers to reasoning that is done after an event has already happened (such as the above page break in this list).
- *ad hoc* means "to this" and refers to reasoning that is specific to an event as it is happening. (Such reasoning is regarded as not being generalizable to other situations.)
- *ad infinitum* means "to infinity' or "without limit".
- *ad nauseam* means "causing sea-sickness" or "to excessive".
- *mutatis mutandis* means "changing what needs changing" or "with the necessary changes having been made".
- *non sequitur* means "it does not follow" and refers to something that is out of place in a logical argument. (This is sometimes abbreviated as "non seq.")
- *Me transmitte sursum, Caledoni!* means, "Beam me up, Scotty!"
- *Quid quid latine dictum sit, altum videtur* means something like, "Anything that is said in Latin will sound profound."

Some References

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